

| UČNI NAČRT PREDMETA / COURSE SYLLABUS                                                                                                               |                           |                                                                   |                              |                                                                                                                                                            |                                         |             |
|-----------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------|-------------------------------------------------------------------|------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------|-------------|
| <b>Predmet:</b>                                                                                                                                     |                           | Numerična aproksimacija in interpolacija                          |                              |                                                                                                                                                            |                                         |             |
| <b>Course title:</b>                                                                                                                                |                           | Numerical approximation and interpolation                         |                              |                                                                                                                                                            |                                         |             |
| <b>Študijski program in stopnja</b><br>Study programme and level                                                                                    |                           | <b>Študijska smer</b><br>Study field                              |                              | <b>Letnik</b><br>Academic year                                                                                                                             | <b>Semester</b><br>Semester             |             |
| Magistrski študijski program<br>Matematika                                                                                                          |                           | ni smeri                                                          |                              | 1 ali 2                                                                                                                                                    | prvi ali drugi                          |             |
| Master's study programme<br>Mathematics                                                                                                             |                           | none                                                              |                              | 1 or 2                                                                                                                                                     | first or second                         |             |
| <b>Vrsta predmeta / Course type</b>                                                                                                                 |                           |                                                                   |                              | temeljni                                                                                                                                                   |                                         |             |
| <b>Univerzitetna koda predmeta / University course code:</b>                                                                                        |                           |                                                                   |                              | M2406                                                                                                                                                      |                                         |             |
| <b>Predavanja</b><br>Lectures                                                                                                                       | <b>Seminar</b><br>Seminar | <b>Vaje</b><br>Tutorial                                           | <b>Klinične vaje</b><br>work | <b>Druge oblike študija</b>                                                                                                                                | <b>Samost. delo</b><br>Individ.<br>work | <b>ECTS</b> |
| 45                                                                                                                                                  |                           | 30                                                                |                              |                                                                                                                                                            | 105                                     | 6           |
| <b>Nosilec predmeta / Lecturer:</b>                                                                                                                 |                           | prof. Emil Žagar, prof. Marjetka Knez                             |                              |                                                                                                                                                            |                                         |             |
| <b>Jeziki / Languages:</b>                                                                                                                          |                           | <b>Predavanja / Lectures:</b> slovenski/Slovene, angleški/English |                              |                                                                                                                                                            |                                         |             |
|                                                                                                                                                     |                           | <b>Vaje / Tutorial:</b> slovenski/Slovene, angleški/English       |                              |                                                                                                                                                            |                                         |             |
| <b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>                                                                        |                           |                                                                   |                              | <b>Prerequisites:</b>                                                                                                                                      |                                         |             |
|                                                                                                                                                     |                           |                                                                   |                              |                                                                                                                                                            |                                         |             |
| <b>Vsebina:</b>                                                                                                                                     |                           |                                                                   |                              | <b>Content (Syllabus outline):</b>                                                                                                                         |                                         |             |
| Aproksimacija funkcij: Izbira prostorov aproksimativnih funkcij. Polinomi. Trigonometrijski polinomi. Odsekoma polinomske funkcije. Stabilnost baz. |                           |                                                                   |                              | Approximation of functions: Spaces of approximation functions. Polynomials. Trigonometric polynomials. Piecewise polynomial functions. Stability of bases. |                                         |             |

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Weierstrassov izrek. Pozitivni operatorji.<br/>Optimalni aproksimativni problem. Eksistenca in enoličnost elementa najboljše aproksimacije. Enakomerna konveksnost, stroga normiranost.</p> <p>Enakomerna aproksimacija s polinomi:<br/>Enoličnost za diskretni in zvezni primer.<br/>Alternacija residuala. Konstrukcija. Prvi in drugi Remesov postopek. Konvergenca. Polinomi Čebiševa. Posplošitve: Čebiševi sistemi funkcij, generalizirani polinomi.</p> <p>Metoda najmanjših kvadratov v zveznem in diskretnem primeru: Ortogonalni polinomi. Tričlenska rekurzivna formula. Gram-Schmidtova ortogonalizacija in numerično stabilnejše izvedbe. Reortogonalizacija. Navezava diskretnega in zveznega primera. Enakomerna konvergenca L2-aproksimacij.</p> <p>Interpolacija: Interpolacija s polinomi. Lagrangeva oblika interpolacijskega polinoma in ostanek. Baricentrična Lagrangeova interpolacija. Deljene difference. Newtonova oblika interpolacijskega polinoma, posplošena Hornerjeva shema. Divergenca interpolacijskih polinomov.</p> <p>Odsekoma polinomske funkcije, zlepki: Eulerjevi poligoni, interpolacija in aproksimacija v drugi normi. Kubični zlepki. B-zlepki kot baza prostora odsekoma polinomskih funkcij. Bézierove krivulje. Zlepki v dveh dimenzijah.</p> | <p>Weierstrass' Theorem. Positive operators.</p> <p>Optimal approximation. Existence and uniqueness of the best approximation. Uniform convexity and strong normed spaces.</p> <p>Uniform approximation by polynomials:</p> <p>Uniqueness in the discrete and continuous case. Iteration of residuals. Construction. The first and the second Remes algorithm. Convergence. Chebyshev polynomials. Generalizations: Chebyshev systems, generalized polynomials.</p> <p>Continuous and discrete least squares:</p> <p>Orthogonal polynomials. Three-term recurrence. Gram-Schmidt orthogonalization, basic and stable version. Reorthogonalization.</p> <p>Connection between discrete and continuous case. Uniform convergence of L2-approximants.</p> <p>Interpolation: Polynomial interpolation. Lagrange form. Barycentric Lagrange interpolation. Divided differences. Newton form and generalized Horner scheme. Divergence of interpolating polynomials.</p> <p>Piecewise polynomial functions, splines: Euler polygons, interpolation and approximation in the second norm. Cubic splines. B-spline bases of piecewise polynomial functions. Bézier curves. Splines in two dimensions.</p> |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

### Temeljni literatura in viri / Readings:

- J. Kozak: Numerična analiza, DMFA-založništvo, Ljubljana, 2008.
- R. L. Burden, J. D. Faires: Numerical Analysis, 8th edition, Brooks/Cole, Pacific Grove, 2005.
- E. K. Blum: Numerical Analysis and Computation : Theory and Practice, Addison-Wesley, Reading, 1998.
- Z. Bohte: Numerične metode, DMFA-založništvo, Ljubljana, 1991.

S. D. Conte, C. de Boor: Elementary Numerical Analysis : An Algorithmic Approach, 3rd edition, McGraw-Hill, Auckland, 1986.

C. de Boor: A Practical Guide to Splines, Springer, New York, 2001.

E. Isaacson, H. B. Keller: Analysis of Numerical Methods, John Wiley & Sons, New York-London-Sydney, 1994.

D. R. Kincaid, E. W. Cheney: Numerical Analysis : Mathematics of Scientific Computing, 3rd edition, Brooks/Cole, Pacific Grove, 2002.

### **Cilji in kompetence:**

Slušatelj dopolni poznavanje analitičnih metod aproksimacije in interpolacije z numeričnimi. Ob domačih nalogah pridobljeno znanje praktično utrdi.

### **Objectives and competences:**

Student supplements knowledge of analytical methods in approximation and interpolation by numerical aspects. By solving homeworks the obtained theoretical knowledge is consolidated.

### **Predvideni študijski rezultati:**

Znanje in razumevanje: Razumevanje pojmov interpolacije in aproksimacije. Praktično obvladanje numeričnih postopkov za konstrukcijo interpolacijskih oziroma aproksimacijskih funkcij.  
Uporaba: Numerična konstrukcija interpolacijskih ali aproksimacijskih funkcij s pomočjo računalnika in ocenjevanje napak na podlagi teorije. Interpolacija in aproksimacija se uporabljata na mnogih področjih, še posebej pri računalniško podprtem grafičnem modeliranju.

Refleksija: Razumevanje teorije na podlagi uporabe.

Prenosljive spretnosti – niso vezane le na en predmet: Spretnost uporabe računalnika pri reševanju matematičnih problemov. Razumevanje razlik med eksaktnim in numeričnim računanjem.

### **Intended learning outcomes:**

Knowledge and understanding: Understanding of interpolation and approximation. Ability of numerical algorithms for construction of interpolating or approximating functions.  
Application: Numerical construction of interpolating and approximating functions using a computer and error estimation based on theory. Interpolation and approximation are used in several fields, in particular in computer aided graphical modelling.

Reflection: Understanding of theory based through applications.

Transferable skills: Skill of using computer for solving numerical problems. Understanding differences between exact and numerical computing.

|  |
|--|
|  |
|--|

|  |
|--|
|  |
|--|

**Metode poučevanja in učenja:**

Predavanja, vaje, domače naloge, konzultacije.

**Learning and teaching methods:**

Lectures, exercises, homeworks, consultations.

**Načini ocenjevanja:**

Delež (v %) /  
Weight (in %)

**Assessment:**

|                                                                                                       |            |                                                                                   |
|-------------------------------------------------------------------------------------------------------|------------|-----------------------------------------------------------------------------------|
| Način (domače naloge, pisni izpit, ustno izpraševanje, naloge, projekt):<br>domače naloge ali project |            | Type (homeworks, examination, oral, coursework, project):<br>homeworks or project |
| pisni izpit                                                                                           |            | written exam                                                                      |
| ustni izpit                                                                                           | 20%        | oral exam                                                                         |
| Ocene: 1-5 (negativno), 6-10 (pozitivno)<br>(po Statutu UL)                                           | 40%<br>40% | Grading: 1-5 (fail), 6-10 (pass) (according to the Statute of UL)                 |

**Reference nosilca / Lecturer's references:**

Marjetka Knez:

- KRAJNC, Marjetka. Geometric Hermite interpolation by cubic  $G^1$  splines. *Nonlinear Analysis, Theory, Methods and Applications*, ISSN 0362-546X. [Print ed.], 2009, vol. 70, iss. 7, str. 2614-2626 [COBISS.SI-ID 15508569]
- KRAJNC, Marjetka. Interpolation scheme for planar cubic  $G^2$  spline curves. *Acta applicandae mathematicae*, ISSN 0167-8019, 2011, vol. 113, no. 2, str. 129-143 [COBISS.SI-ID 16215385]
- JAKLIČ, Gašper, KOZAK, Jernej, KRAJNC, Marjetka, VITRIH, Vito, ŽAGAR, Emil. High order parametric polynomial approximation of conic sections. *Constructive approximation*, ISSN 0176-4276, 2013, vol. 38, iss. 1, str. 1-18 [COBISS.SI-ID 16716121]

Emil Žagar:

- ŽAGAR, Emil. On  $G^2$  continuous spline interpolation of curves in  $\mathbb{R}^d$ . *BIT*, ISSN 0006-

3835, 2002, vol. 42, no. 3, str. 670-688 [COBISS.SI-ID 12027993]

– KOZAK, Jernej, ŽAGAR, Emil. On geometric interpolation by polynomial curves. SIAM journal on numerical analysis, ISSN 0036-1429, 2004, vol. 42, no. 3, str. 953-967 [COBISS.SI-ID 13398617]

– JAKLIČ, Gašper, KOZAK, Jernej, VITRIH, Vito, ŽAGAR, Emil. Lagrange geometric interpolation by rational spatial cubic Bézier curves. Computer Aided Geometric Design, ISSN 0167-8396, 2012, vol. 29, iss. 3-4, str. 175-188 [COBISS.SI-ID 16207449]